$y_{ir} = \beta_0 + \beta_1 x_{1ir} + \cdots + \beta_k x_{kir} + u_{ir}$

$Cov(u_i, u_j) \begin{cases} \neq 0 & i, j \in r \\ = 0 & i \in r, j \in r', \; r \neq r' \end{cases}$

$\sum_{r=1}^{R} \left(\frac{N_r(N_r-1)}{2} + N_r \right) \qquad \frac{N(N-1)}{2} + N$

$R \to \infty$

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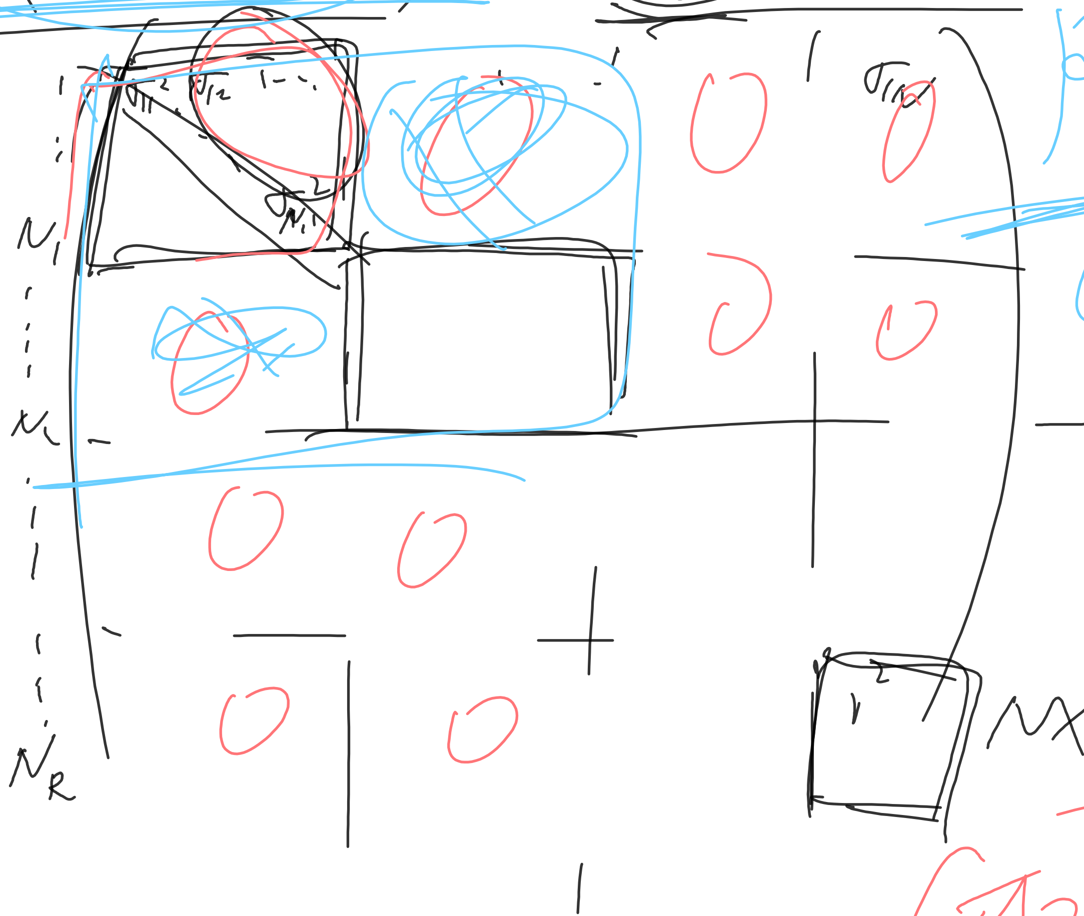

聚类 s.e.

clustered s.e.

$\hat{Y}_i = \hat{\mu}_i(1) + W_i(Y_i(1) - \hat{\mu}_i(1))$

IPW

$$y_{ir} = \beta_0 + \beta_1 x_{ir} + u_{ir}$$

$$\bar{y}_r = \beta_0 + \beta_1 \bar{x}_r + \bar{u}_r$$

$$u_{ir} \sim (0, \sigma^2)$$

$$\bar{u}_r = \frac{1}{N_r} \sum u_{ir} \sim \left(0, \frac{\sigma^2}{N_r}\right)$$

$$\sigma^2 \cdot \frac{1}{N_r}$$

$$\frac{\bar{y}_r}{\sqrt{\frac{1}{N_r}}} = \frac{\beta_0}{\sqrt{\frac{1}{N_r}}} + \beta_1 \frac{\bar{x}_r}{\sqrt{\frac{1}{N_r}}} + \frac{\bar{u}_r}{\sqrt{\frac{1}{N_r}}}$$

$$\sqrt{N_r} \cdot \bar{y}_r = \sqrt{N_r} \cdot \beta_0 + \sqrt{N_r} \cdot \beta_1 \bar{x}_r + \sqrt{N_r}\, \bar{u}_r$$

$$\left(\sqrt{N_r}\, \bar{u}_r\right) = \sqrt{N_r}\, \bar{y}_r - \sqrt{N_r}\beta_0 - \sqrt{N_r}\beta_1 \bar{x}_r$$

$$\min_{(\beta_0, \beta_1)} \sum N_r \bar{u}_r^2 = \min_{\beta_0 \beta_1} \sum_{r=1}^{R} N_r \left(\bar{y}_r - \beta_0 - \bar{x}_r \beta_1\right)^2$$

$$\min \sum W_i \left(y_i - \beta_0 - \beta x_{1i} - \cdots - \beta_k x_{ki}\right)^2$$