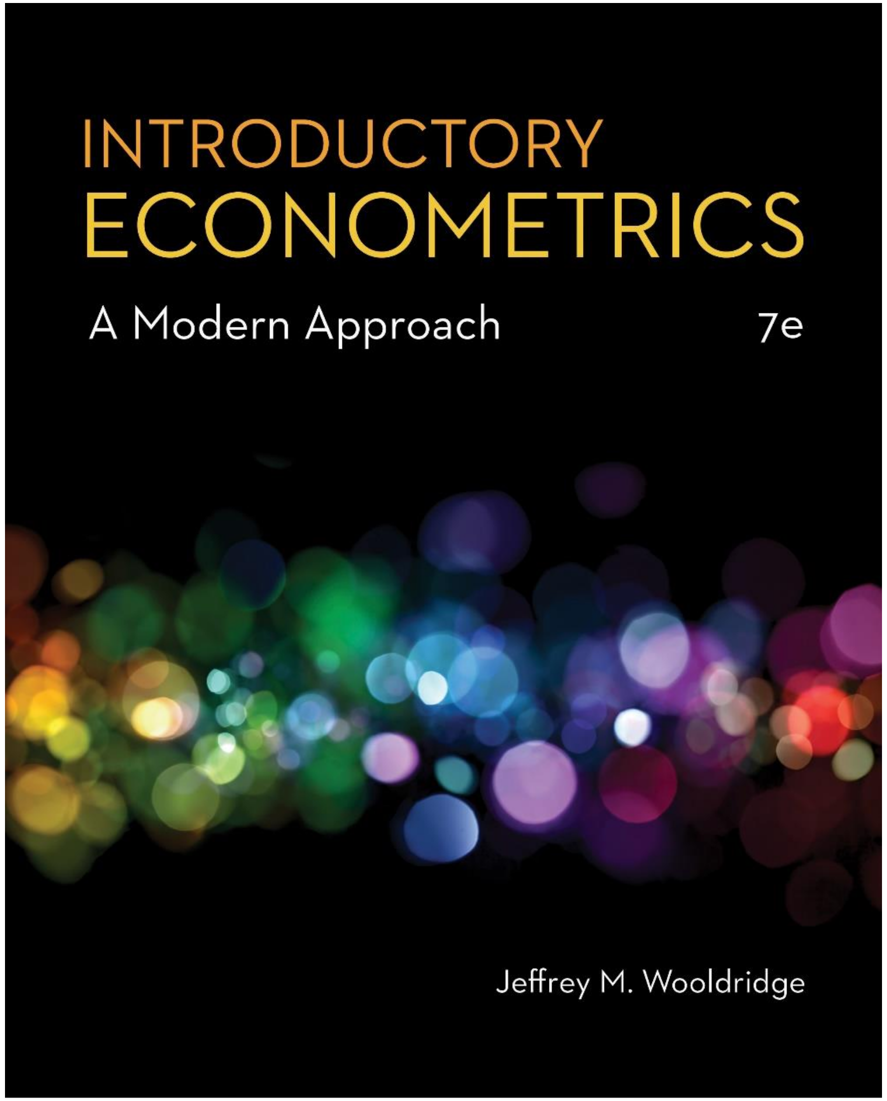
Chapter 5

Multiple Regression Analysis: OLS Asymptotics



Multiple Regression Analysis: OLS Asymptotics (1 of 7)

- So far we focused on properties of OLS that hold for any sample

- Properties of OLS that hold for any sample/sample size
  - Expected values/unbiasedness under MLR.1 – MLR.4
  - Variance formulas under MLR.1 – MLR.5
  - Gauss-Markov Theorem under MLR.1 – MLR.5
  - Exact sampling distributions/tests under MLR.1 – MLR.6

- Properties of OLS that hold in large samples
  - Consistency under MLR.1 – MLR.4
  - Asymptotic normality/tests under MLR.1 – MLR.5
    - Note that we drop MLR.6

MLR.1 (Linear in parameters)
MLR.2 (Random sampling)
MLR.3 (No perfect collinearity)
MLR.4 (Zero conditional mean)
MLR.5 (Homoskedasticity)
MLR.6 (Normality of error terms)

Multiple Regression Analysis: OLS Asymptotics (2 of 7)

- **Consistency**

An estimator θ_n is consistent for a population parameter θ if

$$P(|\theta_n - \theta| < \epsilon) \rightarrow 1 \text{ for arbitrary } \epsilon > 0 \text{ and } n \rightarrow \infty.$$

Alternative notation: $plim\ \theta_n = \theta$ ← The estimate converges in probability to the true population value

- Interpretation:
  - Consistency means that the probability that the estimate is arbitrarily close to the true population value can be made arbitrarily high by increasing the sample size
- Consistency is a minimum requirement for sensible estimators

Multiple Regression Analysis: OLS Asymptotics (3 of 7)

- **Theorem 5.1 (Consistency of OLS)**

$$MLR.1 - MLR.4 \quad \Rightarrow \quad plim\ \hat{\beta}_j = \beta_j, \quad j = 0, 1, \ldots, k$$

- Special case of simple regression model

$$plim\ \hat{\beta}_1 = \beta_1 + Cov(x_1, u)/Var(x_1)$$

One can see that the slope estimate is consistent if the explanatory variable is exogenous, i.e. uncorrelated with the error term.

- **Assumption MLR.4'**

$$E(u) = 0$$
$$Cov(x_j, u) = 0$$

All explanatory variables must be uncorrelated with the error term. This assumption is weaker than the zero conditional mean assumption MLR.4.

Multiple Regression Analysis: OLS Asymptotics (4 of 7)

- For consistency of OLS, only the weaker MLR.4 is needed
- Asymptotic analog of omitted variable bias

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + v$$ ← True model

$$y = \beta_0 + \beta_1 x_1 + [\beta_2 x_2 + v] = \beta_0 + \beta_1 x_1 + u$$ ← Misspecified model

$$\Rightarrow \quad plim\ \tilde{\beta}_1 = \beta_1 + Cov(x_1, u)/Var(x_1)$$

$$= \beta_1 + \beta_2 Cov(x_1, x_2)/Var(x_1) = \beta_1 + \beta_2 \delta_1$$ ← Bias

There is no omitted variable bias if the omitted variable is irrelevant or uncorrelated with the included variable

Multiple Regression Analysis: OLS Asymptotics (5 of 7)

- **Asymptotic normality and large sample inference**
  - In practice, the normality assumption MLR.6 is often questionable
  - If MLR.6 does not hold, the results of t- or F-tests may be wrong
  - Fortunately, F- and t-tests still work if the sample size is large enough
  - Also, OLS estimates are normal in large samples even without MLR.6

- **Theorem 5.2 (Asymptotic normality of OLS)**
  - Under assumptions MLR.1 – MLR.5:

$$\frac{(\hat{\beta}_j - \beta_j)}{se(\hat{\beta}_j)} \overset{a}{\sim} \text{Normal}(0, 1)$$

← In large samples, the standardized estimates are normally distributed

also $plim\ \hat{\sigma}^2 = \sigma^2$

Multiple Regression Analysis: OLS Asymptotics (6 of 7)

- **Practical consequences**
  - In large samples, the t-distribution is close to the Normal (0,1) distribution
  - As a consequence, t-tests are valid in large samples without MLR.6
  - The same is true for confidence intervals and F-tests
  - Important: MLR.1 – MLR.5 are still necessary, esp. homoskedasticity

- **Asymptotic analysis of the OLS sampling errors**

$$\widehat{Var}(\hat{\beta}_j) = \frac{\hat{\sigma}^2}{SST_j(1 - R_j^2)}$$

$\hat{\sigma}^2$: Converges to σ^2

SST_j: Converges to $n \cdot Var(x_j)$

R_j^2: Converges to a fixed number

Multiple Regression Analysis: OLS Asymptotics (7 of 7)

- **Asymptotic analysis of the OLS sampling errors (cont.)**

$\widehat{Var}(\widehat{\beta}_j)$ shrinks with the rate $1/n$

$se(\widehat{\beta}_j)$ shrinks with the rate $\sqrt{1/n}$

- This is why large samples are better
- Example: Standard errors in a birth weight equation

$$n = 1,388 \;\Rightarrow se(\widehat{\beta}_{cigs}) = .00086$$

$$n = 694 \;\Rightarrow se(\widehat{\beta}_{cigs}) = .0013$$

$$\frac{.00086}{.0013} \approx \sqrt{\frac{694}{1,388}}$$

Use only the first half of observations